

OPTIMIZATION OF VESSEL OPERATION TIMES USING ANCHORAGE AREAS: A CASE STUDY OF A BRAZILIAN PORT TERMINAL

USO DE ÁREAS DE FUNDEIO PARA REDUÇÃO DO TEMPO DE MANOBRA DE EMBARCAÇÕES: O CASO DE UM TERMINAL PORTUÁRIO BRASILEIRO

André Andrade Longaray¹ – Universidade Federal do Rio Grande – FURG - longaray@bol.com.br

Paulo Roberto Munhoz - Universidade Federal do Rio Grande – FURG - paulorsmunhoz@hotmail.com

Ricardo Silveira Brasil - Universidade Federal do Rio Grande – FURG - labsadi.furg@gmail.com

Abstract

This study describes the application of an integer programming algorithm for optimizing the maneuver time of vessels operated by a company by using the anchorage area located along the access channel to a port. The research problem occurs because of the need to reduce ships' operating time and the idle times of anchoring areas in the port terminal. The study considers restrictions relating to the estimated times and the execution of maneuvers from beyond the harbor up to the berths, as well as from the anchorage area to the berths. A model is used that considers all the identified constraints to propose anchoring usage for the vessels operated by the companies under review.

Key-Words: Port logistic, anchorage areas, optimization.

Resumo

Este artigo apresenta a sequência de construção de um modelo matemático para a otimização do tempo de manobra de embarcações em um porto marítimo fazendo uso de áreas de fundeio. A questão de estudo surge em razão da necessidade de diminuição do tempo das manobras dos navios e da minimização da ociosidade das áreas de fundeio em um terminal portuário. Foram levadas em conta as restrições relacionando os tempos estimados e de execução das manobras, desde fora de barra até os berços e também da área de fundeio até os respectivos berços. Como resultado, o modelo proposto sugere a utilização das áreas nas manobras das embarcações no terminal marítimo.

Palavras-Chave: logística portuária; áreas de fundeio, otimização.

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¹ Corresponding author.

1 Introduction

A Port development has been used as a strategic element for economic growth in many parts of the world. Its efficiency can be assessed through factors such as operating performance, infrastructure quality, and level of operational safety. The strategy organizations consider effective is one that can minimize the ship's length of stay in its terminal, which is the sum of waiting for berthing, the operation time, and the time until the vessel is released.

The Special Secretariat of Ports (2014) claimed that, by 2015, the infrastructure of Brazilian ports would become insufficient to meet the exponential growth of market demand, with several bottlenecks forecasted in the sector, among which was the generation of queues of ships waiting for their turn to operate. Data published by the Ministry of Development, Industry, and Foreign Trade (MDIC) illustrate that the vessels that passed through Brazilian ports in 2012 were inoperative for almost 90% of the time, as they waited for their turn to dock and unload or receive products (Amato, 2014).

Within this scenario, the Port of the city of Rio Grande moves one of the nation's highest volumes of cargo. Waiting vessels, delays in deliveries to customers, and the consequent increase in operating costs are a reality for this port. This study's unit of analysis is a global company specializing in agricultural products that uses the Rio Grande Port to receive raw materials and ship finished products. As one of the world's largest suppliers of mineral fertilizers, the company is active in more than 40 countries, producing nutritional solutions for the agricultural and farming sector.

This study describes the steps for preparing an integer programming algorithm intended to reduce the total time ships spend in queues upon arrival at Rio Grande Port to unload supplies for the business under study. The article is divided into five sections. Following the introduction, Section 2 presents the literature review upon which the study is based. Section 3 describes the methodology of the research, Section 4 illustrates the proposed optimization algorithm model, and, finally, Section 5 presents the concluding considerations.

2 Theoretical Foundation

Operations research is a multidisciplinary science that studies the development of models in order to aid decision making in complex scenarios. The diversity of these models, with their

characteristics and peculiarities varying according to each critical situation, ensures that success in guiding decision making depends entirely on the correct choice of model.

Participating in a competitive market, managers are increasingly using techniques for building strategies that can be used as a basis for their decisions. Moreover, from a managerial point of view, systematic operational research offers a set of procedures to deal with problems involving the use of scarce resources (Mareth et al., 2012). Companies employ operations research (OR) techniques to support management. Every company must possess an information system, not only as a technological solution but also as a business solution. The methods of OR will meet these expectations because they can be used in all areas, from strategic to operational planning (Chaves, 2011). Furthermore, the model concept is fundamental to OR studies. A model is a simplified representation of reality that aims to interpret the phenomena it models (Chaves, 2011).

According to Hillier and Lieberman (2008), mathematical models of OR have advantages over techniques that are restricted to the verbal description of a problem, as they provide a concise and robust description, resulting in a better understanding of the general structure of the decision-related context and helping to detect significant cause-and-effect relationships. A mathematical programming problem is thus an optimization algorithm in which the objective and constraints are expressed as mathematical functions and functional relationships (Bronson, 1985).

Depending on their nature, these mathematical models can be solved through specific mathematical methods and techniques, including linear programming, dynamic programming, integer programming, nonlinear programming, theory of stocks, theory of lines, simulation, game theory, graph theory, and risk analysis. In mathematical OR models, the representation of a system is usually performed by a set of equations, inequalities, or other mathematical expressions.

Linear programming (LP) is among the mathematical programming techniques most often used for problem resolution within the context of operational research. Linear programming models are used to obtain the optimum value of a given objective function, complying with the specific equations or inequalities representing the model's constraints. What distinguishes this model is that the mathematical functions are expressed in a linear form with regard to both the objective function and the constraints (Souza et al., 2008).

This technique almost always focuses on seeking the best way to allocate scarce resources among competing activities. The allocation problem involves such tasks as scheduling production to maximize profits, mixing the ingredients of a product to minimize costs, selecting an excellent investment portfolio, allocating sales staff in a territory, or defining the cheapest and fastest network of intermodal transport. Linear programming is applied to the programming of decision-making processes for minimal cost and maximum efficiency (Bronson, 1985).

Two mathematical possibilities exist for solving linear programming problems: the algorithms can be solved through the graphical method, whereby, in addition to obtaining the optimal solution, the method allows the OR practitioner to understand the mathematical relationships between the restrictions and their connection with the objective function of the LP model. This method is algebraic, which is sturdier than is the graphical method, because its use does not entail a limitation with respect to the spatial condition of the vectors forming the convex set in the linear model (Longaray, 2013).

An industrial application of LP is seen in oil refineries, which can purchase crude oil from any of several different sources with different compositions and at different prices. Oil can be used to manufacture different products (such as jet fuel, diesel oil, and gasoline) and in varying amounts. The limitations may flow from the restrictions in the amount of crude oil available from a particular source or the refinery's capacity to produce a particular product. A mixture of purchased crude oil and the in-demand manufactured products are what provide maximum profits (Rao, 2009). Some derivations have emerged from among the LP models. Integer programming (IP) is one of them; it is suitable for solving problems involving choices that can be represented by zero-one variables and those involving linear structures with entire, non-integer characteristics (Caixeta-Filho, 2004).

For Colin (2007), IP resulted from a limitation found in LP, when the use of integer variables in problem solving was needed. The use of IP and LP entail similar levels of problem-solving difficulty; however, as the latter's space for solutions is infinite, while the space of IP solutions is finite, IP solutions tend to be more reliable (Santos et al, 2012).

An IP problem can thus be seen as a LP problem with the restriction that the variables have to be integers; this concept involves a relaxation in LP. According to Colin (2007), relaxation in the LP of an IP problem is an IP problem without the restriction that the variables must be integers.

Integer programming problems can be classified as follows: pure integer programming problems (in which all variables are integers and generic); mixed integer programming problems (where parts of the variables are integers and parts are continuous); and integer programming problems with 0–1 variables—where all the variables assume the values of 0 or 1 (Colin, 2007).

According to Longaray and Damas (2013), IP models are potentially valuable because many of the practical problems faced by organizations involving activities and resources such as machinery, ships and operators are indivisible. Furthermore, most optimization problems of a combinatorial nature can be formulated as integer programs.

Recent studies have demonstrated the applicability of the integer program in finding solutions to organizational problems in different fields of business. Steiner et al. (2009) used IP to improve the production of industrial paper. Souza et al. (2011) used mixed IP to build an algorithm for waste management on offshore drilling rigs. Other applications involve cost management (Mareth et al., 2012) and cargo transport logistics (Santos et al, 2012).

Against this background, this paper proposes an IP algorithm that minimizes the maneuver time of vessels operated by the company under study, restricted by the legal and physical limits of the activity. It is expected that the time between vessel movements will be reduced, thus reducing the idle time in the port terminal.

3 Methodological Procedures

The methodology developed in this work deals with the model proposed by Roesch (2010), in which the methodological procedures are described in terms of the research's purpose, characteristics, design, data collection techniques, and analysis.

Concerning purpose, among the alternatives (Roesch, 2010), this paper is applied research with qualitative and quantitative characteristics. This study takes a predominantly quantitative approach to the evaluation of the results and a more qualitative approach to its formative assessment of idle times in the terminal. Preparing an optimization model appropriate to this study required observing vessel arrivals and analyzing the times of the maneuvers from beyond the harbor to the north/south berths and from the anchorage area to the respective berths.

The organization had only two berths for mooring ships, North and South, which could handle two ships simultaneously. This study uses the case study method to achieve results that meet the needs of the organization and the customers. According to Roesch (2010), case study methods differ from historical methods in examining the present rather than the past.

The study's analysis followed the mathematical treatment of the data, enabling the construction of the IP algorithm and the preparation of the objective function for minimizing the maneuver time of the company's vessels as well as the restrictions highlighted in the context of these movements.

4 Proposed IP Algorithm

This section describes the steps in the proposed IP algorithm, featuring a sequential presentation of the problem's elaboration process, up to the execution of the algorithm and analysis of the results.

4.1 Types of vessels operated by the company under study

Bulk carriers are intended solely for transporting bulk materials. Their holds, without divisions, have rounded corners, facilitating cargo stowing. Most of these vessels operate as tramps (i.e., without regular lines) and transport low-value goods.

Cargo ships are built for general cargo transport. Their holds are usually divided horizontally, forming what are called "shelves" (or "decks"), where diverse types of cargo can be accommodated for transportation. They are still used in some regular traffic; they offer a regular service, both chartered and not, and at a speed suitable for their operations (Goebel, 1996).

However, tankers are exclusively intended for the transport of bulk liquids. The current trend is to use a fleet of more economical and agile vessels.

4.2 Anchoring areas

An anchoring area can be understood as equivalent to a mooring or anchorage—a place where a vessel casts anchor when approved by the responsible agencies.

The areas used for mooring at the Organized Port of Rio Grande are regulated and administered by the Port Authority, in both its official statements and in the safety of navigation, through the anchoring permits provided by the port administration. Seven anchorage areas are available. Each site has different restrictions regarding footage and the use of vessels. The specific study area is one of the few authorized maneuvering areas. It enjoys a privileged position for the organization in question, being located 300 meters from the company. Anchoring at this site is permitted for ships up to 190 meters long and with a maximum draft of 9.15 meters (32.25 feet), according to the conditions determined by the service orders.

4.3 Problem formulation

From the moment ships begin to carry the load stipulated by their contract until their berthing, the agencies appointed for such operations are responsible for communication, passing along various estimates and events related to the vessel to everyone involved until the arrival of the ship at the entry harbor to the Rio Grande Port–Rio Grande do Sul), and announcing the notice of readiness to all with an interest in the cargo.

For this announcement, a count is begun of the time the vessel is available to the company, regardless of the availability of mooring berths. The time for each vessel is determined according to the amount and type of raw stock established by the contract. A few hours of ship delays in ports can generate large economic losses for the organizations involved, since the company under study often carries out operations for third parties. Thus, the results of its operations can expose its clients to injury, while having vessels ahead of schedule can benefit its customers with positive returns.

No goals are pursued during idle time in the port terminals of the city of Rio Grande–Rio Grande do Sul. For the organization under review, this results in an average of three hours of idle time per operation for the terminal, with an average annual flow of 140 ships. This study seeks to minimize the maneuver times of the vessels operated by the company under study while respecting the applicable legal and physical limits. Reducing the spaces between the vessels' movements will also reduce the port terminal's downtime.

Figure 1 outlines the steps in the vessels' mooring maneuvers, from the time they reach the harbor until they dock at the private port terminal.

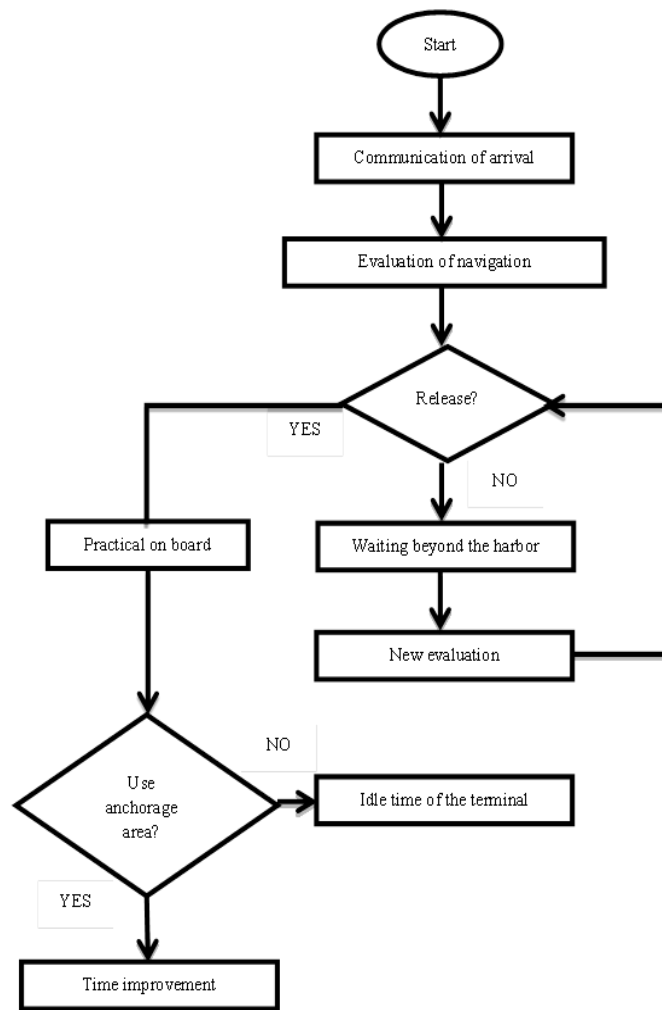


Figure 1: *The decision-making process during berthing* (source: authors' own work)

Based on the Figure 1, a model must be developed that aggregates all the study requirements. An algorithm has been developed that covers the times of the vessels' maneuvers during the study period: the average maneuver time from beyond the harbor to the North and South berths during the study period as estimated by the pilot of the Rio Grande Harbor–Rio Grande do Sul and the maneuver time as estimated by the harbor pilot from the area to the North and South berths.

During the study period, the organization conducted operations using 28 vessels of all kinds: bulk ships (NT1), general cargo ships (NT2), and tankers (NT3). The tankers do not operate in the

South berth due to the need for liquid discharge connection lines; these vessels therefore discharge in the North berth. Table 1 shows the ships' total waiting times.

Table 1: *Ship waiting times*

Total waiting times by type of ship during the reporting period (in hours)			
Type	Bulk	General Cargo	Tanker
Berth 01	1824	259	235
Berth 02	2068	0	-

Source: Research data

The values above represent how long each type of vessel waited beyond the harbor (in hours) until mooring began. These numbers represent how long the ships waited until starting their operations during periods of intense movement. These data illustrate how significant this study is to everyone involved, given the delays throughout the chain. Table 2, presenting the average time of the vessels' maneuvers, is not used in the algorithm but rather serves to demonstrate the feasibility of using this anchorage area.

Table 2: *Average time of the maneuvers*

Average time of the maneuvers for the study period (July/August; in minutes)			
Type	Bulk	General Cargo	Tanker
Berth 01	109	88	103
Berth 02	108	0	-

Source: Research data

The results shown in Table 2 were supplied by the harbor pilot at the Rio Grande Port–Rio Grande do Sul and by the organization under study, based on the history of all the vessel times (where it was possible to identify all the operations), measured as time until docking. The average time for each type of vessel that operated in these two months was determined based upon information such as the shipping time for a professional of this type of service from mooring until landing. Table 3 presents the time for each maneuver area.

Table 3: *Time for each maneuver area*

Maneuver time of the ships beyond the harbor to the berths (in minutes)			
Type	Bulk	General Cargo	Tanker
Berth 01	85	85	85
Berth 02	82	0	-

Source: Research data

Informal conversations and reports provided by SUPRIG and the pilot of the harbor suggest that the times in Table 3 would be ideal and legal for each ship maneuver from beyond the harbor to each berth, according to the applicable standards or targets. No support is offered recipients when a vessel takes an hour or three hours to complete its operations. Table 4 presents the average times from the anchorage area to the berths for docking the ships.

Table 4 Average times from the anchorage area to the berths

Average maneuver times for the ships from the area to the berths (in minutes)			
Type	Bulk	General Cargo	Tanker
Berth 01	40	40	40
Berth 02	25	25	-

Source: Research data

The times shown in Table 4 are the estimated times the ships spend in operations from their entry until their berthing. This information was provided by the pilot of the harbor according to tests conducted during the study preparation period after a request was submitted in conjunction with the ships' agents. The anchorage area was not used by ships not in compliance with the site's footage and draft restrictions.

The analysis of the algorithm uses units in minutes to facilitate the understanding and standardization of the study modeling, from the preparation of the objective function to the constraints identified for this study. The features listed in this analysis allow the formulation of a problem in integer LP designed to minimize the vessels' length of stay.

The algorithm was developed to reduce the vessels' maneuvering time and identify how much the non-use of this anchorage directly impacts the organization under study. How the organization was affected by this non-use was then compared with the use of this area.

We therefore have the following integer LP model. Objective function:

$$\text{Min (T)} = 13\text{TMNT11} + 1\text{TMNT12} + 2\text{TMNT13} + 8\text{TMNT21}$$

where:

TMNT11= Operating time of ships unloading in Berth 01 of Type 01;

TMNT12= Operating time of ships unloading in Berth 01 of Type 02;

TMNT13= Operating time of ships unloading in Berth 01 of Type 03;

TMNT21= Operating time of ships unloading in Berth 01 of Type 01.

Data for July and August 2012 were analyzed because this covered the period during which vessel wait times caused concern due to the increased demand forecasted for the coming years.

Restrictions:

a) The time estimated by the harbor pilot for the maneuvers of the vessels unloading in Berth 01 of Type 01, 02, and 03 must be greater than or equal to 40 minutes (the minimum time allowed for the

use of the anchoring area), with a maximum of 85 minutes provided for the execution of maneuvers as estimated by the harbor pilot at the Rio Grande Port–Rio Grande do Sul with no use of this area. Therefore, the following is true:

$$TMNT11 \geq 40$$

$$TMNT11 \leq 85$$

$$TMNT12 \geq 40$$

$$TMNT12 \leq 85$$

$$TMNT13 \geq 40$$

$$TMNT13 \leq 85$$

b) The operating time estimated by the harbor pilot for ships unloading in Berth 02 of Type 01 must be greater than or equal to 25 minutes (the minimum time allowed for the use of the anchoring area) and with a maximum time of 82 minutes provided for the execution of maneuvers as estimated by the harbor pilot at the Rio Grande Port–Rio Grande do Sul with no use of the area:

$$TMNT21 \geq 25$$

$$TMNT21 \leq 82$$

c) Below is the non-negativity constraint of the study:

$$TMNT11 \geq 0$$

$$TMNT12 \geq 0$$

$$TMNT13 \geq 0$$

$$TMNT21 \geq 0$$

d) The anchorage area under study has limitations on the footage and draft of the vessels (i.e., depth of the vessels, in feet); thus, this space may be used only according to the pre-established

specifications. A ship must not exceed 190 meters in length and 32.25 feet in average draft. Vessels that exceed these limits are not accounted for in the algorithm.

4.4 Resolution of the algorithm

The algorithm was resolved using the Solver equation optimization and solution tool, available in the Microsoft Excel 2010 software package, and LP SOLVE [19], the tool recommended for solving IP problems, was used to review and check the results. The results were prioritized in Solver; the defined resolution method was LP SIMPLEX, using the GRG mechanism. Among the alternatives provided by this tool, the resolution of the algorithm was taken into consideration, as well as the determination of the limits and the determination of the minimum and maximum times for both berths. Windows 7 Ultimate was used to implement the algorithm, with an Intel (R) Core TM i5-2430M processor. The version of LP SOLVE employed was 5.5.2.0.

4.5 Analysis of the results

Of the 28 ships that operated during the analysis period, only 24 were taken into consideration due to restrictions on the length and draft in the anchorage area. The study excluded three bulk carrier vessels, two of which operated in the northern berth and one in the southern berth, and a general cargo ship that operated in the southern berth. The analyses used the information shown in Table 5.

Table 5: *Movement of vessels*

Movement of the vessels during July/August						
	NT1		NT2		NT3	
Type	no. of vessels	Average Maneuver time	no. of vessels	Average maneuver time	no of vessels	Average maneuver time
Berth 01	13	109	1	88	2	103
Berth 02	8	108	-	-	-	-

Source: Research data

Based on the lower limit (T_{min}), the time stipulated for the operation of the vessels from the anchorage area to Berth 01 or 02, and the upper limit (T_{max}), the operating time of the vessels from beyond the harbor to the North or South berth, and the average time of the movements of each type of ship (TMMNT), the objective function and its restrictions were defined as shown in Table 6 as follow.

Table 6: *Data for resolution by Solver*

Vessels	Quantity	TMMNT	Tmin.	Tmax.	Total
NT11	13	109	40	85	1417
NT12	1	88	40	85	88
NT13	2	103	40	85	206
NT21	8	108	25	82	864
Total quantity of vessels					24
Total time (in minutes)					2575

Source: Research data

The data presented in the execution of the algorithm indicate that the resulting values suggest the use of the anchorage area in all maneuvers as long as the vessels comply with the restrictions, as shown in Table 7.

Table 7: *Data produced by Solver*

Vessels	Quantity	TMMNT	Tmin.	Tmax.	Total
NT11	13	40	40	85	520
NT12	1	40	40	85	40
NT13	2	40	40	85	80
NT21	8	25	25	82	200
Total quantity of vessels					24
Total time (in minutes)					840

Source: Research data

The results allow a comparison of the total times for the maneuvers of ships of each type in the period under study, illustrating how important using the anchorage area is in modifying the results for the organization. The bulk ships operating in Berth 01 displayed a reduction of 897 minutes during the period, a saving of 63% of the operating time. The general cargo ships operating in Berth 01 reduced operations by 48 minutes, 54% of the time taken. The tankers displayed a reduction of 126 minutes, a 61% time saving. Finally, the bulk vessels operating in Berth 02 saved 664 minutes, a 77% reduction in maneuver time. These data are illustrated in Table 8.

Table 8: Results of each type of vessel in Solver

Cell	Name	Original value	Final value
\$C\$4	NT11 TMM NT	1417	520
\$C\$5	NT12 TMM NT	88	40
\$C\$6	NT13 TMM NT	206	80
\$C\$7	NT21 TMM NT	864	200
\$F\$11	Total time (minutes)	2575	840

Source: Research data

Using the anchoring area would clearly allow the company under study to improve the maneuver time of the vessels operating in its private port terminal: the organization would save 1735 minutes, a 67% time reduction (or 29 hours of optimized maneuver time), time during which its port terminal is idle, when no operations occur, and when its customers are waiting. These results indicate potential benefits not only for the company but also for the general public: faster delivery to customers would allow them to cultivate their lands sooner, enabling them to provide products to the consumer markets more quickly, thus increasing the turnover of the products in circulation.

5 Conclusion

This study analyzes the length of stay of ships in a port terminal and uses an optimization model in order to reduce the time needed to maneuver vessels through the anchoring area. Tools

used in OR were employed, and the definition of the objective function along with its respective restrictions were considered.

The adaptation of the formulated model determined the choice of the computational tool used to process the data obtained in the research. As tests have used other software tools, such as GRASP, GAMS, and MPL, the decision was made to work with Solver (from the Microsoft Excel 2010 package) and LP Solve because of their ease of understanding and their assistance in describing the results.

Through the optimization model, simulations were conducted comparing the current and proposed scenarios based on an analysis of the two busiest months from 2008 to 2012 in order to identify the terminal's idle time and propose the use of the anchoring area, thereby reducing the costs directly related to these events and allowing the company to offer better and more flexible service to its customers.

The algorithm considered only certain times among a number of different times identified during the course of the survey. The explicit information in this study was made available through numerous reports and informal conversations with the planning sector of the organization under study, with the department of statistics and monitoring at the Rio Grande Port–Rio Grande do Sul, as well as with the harbor pilot.

The company under study paid US \$1,377,122.09 in costs in 2012 alone, 20.87% of which represented waiting ships, illustrating how significant this research is for the organization. The competitive advantage of the port in Rio Grande–Rio Grande do Sul consists of its anchorage areas, with their restrictions. However, little use is made of this advantage. It conducts its operations as it always has, without innovating, taking risks, or demanding better services and thus accumulating over 20 vessels beyond the harbor during the harvest season.

A limitation of this study is that vessels longer than 190 meters and with a draft greater than 32.25 feet were prevented from using the anchorage area and had to perform their maneuvers along the lines currently employed due to the site's restrictions.

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